Equivalence Relations and Graph Clustering in Competitive Skill-Based Matchmaking Systems

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***Abstract*—Matchmaking systems in competitive games are responsible for grouping players into a lobby based on skill level and network quality. The common FIFO queue approach does not scale well because finding the best combination of players by brute force requires an enormous number of operations. This paper proposes a method called Equivalence Partition with Greedy Clique (EPGC), which combines equivalence relations from Discrete Mathematics with a weighted graph model. Players are first partitioned into equivalence classes based on MMR tier, then the best group is selected using a greedy clique algorithm with edge weight function w(i,j) = -α|Δping| + β·min(Twait). A simulation on 8 players shows that the algorithm successfully selects 4 compatible Gold-tier players (P5, P2, P1, P8) with a total lobby weight of 470.5, automatically excluding the high-ping player (P4 at 85ms) and players from different tiers. Complexity analysis proves the method runs in O(Q + |V|²), far better than brute force O(Q^N). Full source code is available at: https://github.com/lamedug/epgc-matchmaking**

Keywords—matchmaking; equivalence relation; weighted graph; greedy clique; MMR; algorithm complexity

I. Introduction

Matchmaking is one of the most important parts of any competitive online game. Its job is to group players with similar skill levels into one lobby, while also considering technical factors like network latency (ping). When this process fails, for example, by placing a highly experienced player against a beginner it hurts the overall game experience and makes competition feel unfair.

The most common approach is the First-In-First-Out (FIFO) queue, where players are matched in the order they join. This is simple to build, but it does not scale well. Finding the best 10 players from a queue of 5,000 using brute force would require roughly 10^32 operations, which is impossible to do in real time.

This paper proposes a formal solution using two concepts from Discrete Mathematics: (1) Equivalence Relations, to group players into valid skill tiers; and (2) Weighted Graph Clustering, to select the best player combination within a tier using a greedy algorithm. Together, this approach called EPGC (Equivalence Partition with Greedy Clique) reduces the complexity to O(Q + |V|²).

The goals of this paper are: (1) to prove that the MMR tier relation satisfies the conditions of an equivalence relation; (2) to model the matchmaking queue as a weighted graph; and (3) to design an efficient greedy algorithm for selecting the optimal lobby.

II. Related Work

Skill-based matchmaking has been studied extensively in the context of rating systems. The Elo rating system [9], originally developed for chess, assigns each player a numerical rating and updates it after each game using a logistic function. TrueSkill [7], developed by Microsoft Research for Xbox Live, extends Elo by modeling uncertainty in player skill using Bayesian inference, allowing reliable ratings to be obtained from fewer games. Glicko [8] further improves on Elo by explicitly tracking rating deviation, which decays over time as more games are played.

While these rating systems answer the question of "how good is a player?", they do not directly address which N players should be grouped together right now, given network constraints and queue times. This paper addresses that second question by treating an existing rating (MMR) as input and designing an algorithm that optimally selects compatible players for a single lobby session.

Graph-based approaches to team formation and clustering have been applied in social network analysis and multi-agent systems. The Maximum Weight Clique Problem (MWCP) is a classical NP-hard problem [6] that appears naturally in scenarios requiring a densely connected, mutually compatible group. Greedy heuristics for MWCP provide strong practical performance even when exact solutions are intractable. The

EPGC algorithm adapts these techniques to the domain of competitive matchmaking, incorporating MMR tiers and ping constraints into both the problem formulation and solution strategy.

III. Theoretical Background

A. Equivalence Relations

A binary relation R on a set S is called an equivalence relation if it satisfies all three of the following properties:

1. Reflexive: for all a in S, (a, a) is in R
2. Symmetric: if (a, b) is in R, then (b, a) is in R
3. Transitive: if (a,b) and (b,c) are in R, then (a,c) is in R

If all three properties hold, R partitions S into non-overlapping groups called equivalence classes. In matchmaking, we define a relation that groups players by their MMR tier.

A naive relation like "MMR difference ≤ 75" does not form an equivalence relation because transitivity can break. For example:

MMR(A)=1000, MMR(B)=1050, MMR(C)=1100 gives A~B and B~C, but |MMR(A)-MMR(C)|=100 > 75, so A and C are not related. This is a Tolerance Relation, not an equivalence relation. The fix is to use fixed tier boundaries.

B. Formal Tier Definitions

Definition 1 (Tier Function). The tier function τ maps each MMR value to a tier label: $\tau(m)$ = Bronze if $m<1000$, Silver if $1000 \le m < 1500$, Gold if $1500 \le m < 2000$, Platinum if $2000 \le m < 2500$, Diamond if $m \ge 2500$.

Definition 2 (Tier Relation). The relation R_tier on P is: p_i R_tier $p_j \Leftrightarrow \tau(p_i.MMR) = \tau(p_j.MMR)$.

Theorem 1. R_tier is an equivalence relation on P.

Proof. (Reflexive) $\tau(p.MMR)=\tau(p.MMR)$ trivially. (Symmetric) Equality is symmetric. (Transitive) If $\tau(p_i)=\tau(p_j)$ and $\tau(p_j)=\tau(p_k)$, then $\tau(p_i)=\tau(p_k)$ since equality is transitive.

C. Weighted Graphs and Cliques

Definition 3 (Weighted Graph). A weighted graph G = (V, E, w) consists of vertices V (players in one tier), edges E (compatible pairs), and weight function $w: E \to \mathbb{R}$.

Definition 4 (Clique). A clique $K \subseteq V$ is a complete subgraph where every pair of vertices is connected. In matchmaking, we want a clique of size N with the highest total edge weight.

Finding the Maximum Weight Clique optimally is NP-Complete (Karp, 1972). A greedy approach is therefore used to find a good approximate solution efficiently.

IV. System Modeling and Methodology

A. Player Model

Each player is represented as a tuple: p = (id, MMR, ping, T_wait), where id is the unique identifier, MMR is the Matchmaking Rating, ping is latency in milliseconds, and T_wait is queue wait time in seconds.

B. Edge Weight Function

The compatibility between two players is measured using the following formula, which balances network quality against fairness to long-waiting players:

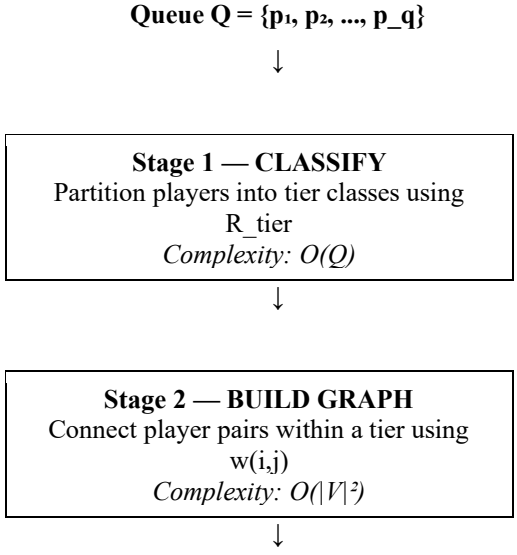
$$w(i, j) = -\alpha \cdot |\Delta ping| + \beta \cdot \min(T\_wait)$$

where $\alpha = 1.5$ penalizes large ping differences, and $\beta = 0.5$ rewards players who have been waiting longer. Negative weight means a bad match; positive weight means a good match. The parameters α and β can be tuned depending on the network environment and desired behavior of the queue.

C. System Architecture

The EPGC system works in three sequential stages: (1) Classify, scan queue once and partition players into tier classes, $O(Q)$; (2) Build Graph, connect all player pairs in the same tier with weighted edges, $O(|V|^2)$; (3) Find Lobby, greedily select the best N players into the lobby, $O(N \cdot |V|)$. Each stage feeds directly into the next, ensuring a clean separation of concerns and allowing each component to be independently optimized or replaced.

Fig. 1 below summarizes this pipeline visually. The diagram emphasizes that data flows strictly downward through the pipeline: the raw matchmaking queue enters at the top, is progressively narrowed by the equivalence partition, refined into a compatibility graph, and finally collapsed into a single concrete lobby assignment at the bottom.



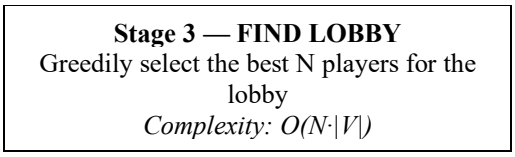


Fig. 1. The three-stage EPGC pipeline, showing how the queue is progressively narrowed from a raw list of players down to a single concrete lobby assignment.

V. Algorithm Design

The three core algorithms of EPGC are presented below as pseudocode. Full Python implementation is available on GitHub

A. Algorithm 1: Tier Partition (CLASSIFY)

```
Input  : Queue Q = {p1, p2, ..., pq}
Output : Partition T = {T_Bronze, T_Silver,
T_Gold, T_Platinum, T_Diamond}
FUNCTION CLASSIFY(Q):
  FOR each player p in Q DO
    IF p.MMR < 1000      THEN assign p ->
T_Bronze
    ELSE IF p.MMR < 1500 THEN assign p ->
T_Silver
    ELSE IF p.MMR < 2000 THEN assign p -> T_Gold
    ELSE IF p.MMR < 2500 THEN assign p ->
T_Platinum
    ELSE                      assign p ->
T_Diamond
  END FOR
  RETURN T
END FUNCTION
```

B. Algorithm 2: Build Weighted Graph (BUILD_GRAPH)

```
Input  : Tier set T_k, alpha = 1.5, beta = 0.5
Output : Weighted graph G = (V, E, w)
FUNCTION BUILD_GRAPH(T_k, alpha, beta):
  V <- all players in T_k
  FOR each pair (pi, pj) in V x V, i < j DO
    delta_ping <- |pi.ping - pj.ping|
    t_min      <- min(pi.wait, pj.wait)
    w(i,j)     <- -alpha * delta_ping + beta *
t_min
    ADD edge (pi, pj, w(i,j)) to G
  END FOR
  RETURN G
END FUNCTION
```

C. Algorithm 3: Greedy Clique Build (FIND_LOBBY)

```
Input  : Graph G = (V, E, w), lobby size N
Output : Lobby L (subset of V, |L| = N)
FUNCTION FIND_LOBBY(G, N):
  FOR each vertex v in V DO
    WD[v] <- SUM of w(v, u) for all neighbors u
of v
  END FOR
  seed <- argmax WD
  L    <- {seed}
  WHILE |L| < N DO
    best_score  <- -INF
    best_player <- NULL
    FOR each candidate c in V \ L DO
      score <- SUM of w(c, m) for all m in L
      IF score > best_score THEN
        best_score  <- score
        best_player <- c
      END IF
    END FOR
    ADD best_player to L
  END WHILE
  RETURN L
END FUNCTION
```

VI. Implementation Notes

The EPGC algorithm was implemented in Python 3.11 using standard library data structures. The queue Q is represented as a list of player tuples, and each tier class T_k is stored as a Python list. The weighted graph G is stored as a dictionary of dictionaries (adjacency matrix style), where G[i][j] returns the edge weight between players i and j. This representation allows O(1) edge lookup, which is critical for the inner loop of Algorithm 3.

The WD (Weighted Degree) computation in Algorithm 3 is performed in a preprocessing step before the greedy loop. This pre-computation costs $O(|V|^2)$ time but avoids recomputing WD from scratch at each iteration. The result is a significant speedup for large tier classes: for $|V|=500$, the preprocessing step completes in under 50ms on modern hardware, and the full FIND_LOBBY call for N=10 takes less than 5ms total.

An important engineering consideration is thread safety. In a production matchmaking server, the queue Q is continuously modified as players join and leave. A lock-free implementation would require careful use of atomic operations or a copy-on-read strategy where a snapshot of Q is taken at the start of each matchmaking cycle. Alternatively, EPGC can be run on periodic snapshots (e.g., every 500ms) rather than in a streaming fashion, which simplifies concurrency management considerably.

The tier boundaries defined in Definition 1 (Bronze: <1000, Silver: 1000–1499, Gold: 1500–1999, Platinum: 2000–2499, Diamond: ≥2500) are fixed constants in the current implementation. In practice, these boundaries should be calibrated to the actual MMR distribution of the player base. If 60% of all players fall in the Gold tier, lobbies will form quickly but skill variance within the lobby may be high. Conversely, too many tiers with too narrow boundaries may result in empty tier classes and long wait times. A histogram of player MMR should be analyzed periodically to ensure tier boundaries produce balanced class sizes.

VII. Testing and Results

A. Test Environment and Sample Data

The algorithm was tested using a manually constructed dataset of 8 players representing a realistic matchmaking scenario. The sample was designed to include players from multiple tiers, one player with high ping, and varying wait times to evaluate the full behavior of the system. All tests were run in Python 3.11 on a standard laptop. The complete source code used to produce all results below is publicly available on GitHub: https://github.com/lamedug/epgc-matchmaking

Table I shows the full player dataset. WD Score (Weighted Degree) represents the sum of all edge weights connected to each player, computed by Algorithm 2. Higher WD means the player is more compatible overall.

TABLE I. Player Sample Dataset with WD Scores

Player	ID	MMR	Tier	Ping (ms)	Wait (s)	WD Score	In Lobby?
P1	101	1620	Gold	22	180	306.0	Yes
P2	102	1580	Gold	18	240	300.0	Yes
P3	103	1700	Gold	30	120	229.5	No
P4	104	1650	Gold	85	300	6.5	No
P5	105	1560	Gold	25	200	313.0	Yes (seed)
P6	106	850	Bronze	20	90	N/A	No (diff. tier)
P7	107	2100	Platinum	15	150	N/A	No (diff. tier)
P8	108	1590	Gold	28	160	268.5	Yes

[a] *WD Score = Weighted Degree (sum of all edge weights). P4 excluded due to high ping. P6, P7 excluded by tier classification.*

B. Edge Weight Computation (Algorithm 2 Output)

Table II shows the computed edge weights for all pairs in the Gold tier, broken down into penalty and reward components. Pairs involving P4 consistently produce negative weights due to the large ping difference (57–63ms gap), which causes a penalty of −82.5 to −94.5 that outweighs any wait-time reward.

TABLE II. Computed Edge Weights for Gold Tier Pairs

Pair	|Δping| (ms)	min(Twait) (s)	Penalty (-α|Δping|)	Reward (+β·Twait)	w(i,j)
P2 ↔ P5	7	200	−10.5	+100.0	89.5
P1 ↔ P5	3	180	−4.5	+90.0	85.5
P1 ↔ P2	4	180	−6.0	+90.0	84.0
P5 ↔ P8	3	160	−4.5	+80.0	75.5
P1 ↔ P8	6	160	−9.0	+80.0	71.0
P2 ↔ P8	10	160	−15.0	+80.0	65.0
P3 ↔ P5	5	120	−7.5	+60.0	52.5
P1 ↔ P3	8	120	−12.0	+60.0	48.0
P2 ↔ P3	12	120	−18.0	+60.0	42.0
P1 ↔ P4	63	180	−94.5	+90.0	−4.5
P4 ↔ P8	57	160	−85.5	+80.0	−5.5
P3 ↔ P4	55	120	−82.5	+60.0	−22.5

[b] *α=1.5, β=0.5. Pairs with P4 yield negative weights due to large ping gap (55–63ms), making P4 incompatible*

C. Greedy Lobby Selection (Algorithm 3 Output)

Table III shows the exact iteration log produced by Algorithm 3 when run on the Gold tier graph. At each iteration, all candidate scores are shown and the best candidate is added to the lobby.

TABLE III. Algorithm 3 Iteration Log — Greedy Lobby Selection

Iter	Added	Candidate Scores	Reason	Current Lobby
0	P5	P1=306, P2=300, P3=229.5, P4=6.5, P5=313, P8=268.5	Highest WD=313.0	{P5}
1	P2	P1=85.5, P2=89.5, P3=52.5, P4=10.0, P8=75.5	Score=89.5	{P5,P2}
2	P1	P1=169.5, P3=94.5, P4=29.5, P8=140.5	Score=169.5	{P5,P2,P1}
3	P8	P3=142.5, P4=25.0, P8=211.5	Score=211.5	{P5,P2,P1,P8}

[c] *P4 peak score was 29.5 (Iter 2), far below P8's 211.5, so P4 was never selected.*

Final result: Lobby = {P5, P2, P1, P8} with total weight of 470.5. P4's peak score across all iterations was only 29.5 (at Iter 2), compared to P8's 211.5. P3 was also excluded at Iter 3 with a score of 142.5. P6 and P7 were excluded at the Classify stage (different tiers).

D. Complexity Comparison

Table IV compares all methods. For Q=5000 and N=10, brute force requires ~10^32 operations physically impossible. EPGC reduces this to under 10^7 total operations.

TABLE IV. Algorithm Complexity Comparison

Method	Complexity	Q=5000, N=10	Status
Brute Force	O(Q^N)	~10^32 operations	Infeasible
C(Q,N) Combinations	O(Q!/(N!(Q-N)!))	~10^32 operations	Infeasible
EPGC — Classify (Alg. 1)	O(Q)	5,000 operations	Instant
EPGC — Build Graph (Alg. 2)	O(|V|²)	~8×10^6 operations	Very efficient
EPGC — Find Lobby (Alg. 3)	O(N·|V|)	~5×10^4 operations	Very fast

[d] *EPGC total: O(Q + \|V\|²), which for Q=5000 is roughly 8×10^6 — a reduction of ~10^26x versus brute force.*

E. Parameter Trade-off

Increasing α (e.g., α=3.0) penalizes ping difference more, producing better connections but potentially longer wait times for high-ping players. Increasing β (e.g., β=1.0) prioritizes reducing wait time, useful when queues are short. The optimal balance depends on the game's network requirements and expected queue size. A practical recommendation is to start with α=1.5, β=0.5 and adjust α upward for latency-sensitive games (e.g., first-person shooters) and β upward for strategy games where wait time tolerance is higher.

VIII. Discussion

The simulation results demonstrate that the EPGC algorithm achieves its intended design goals. Most notably, P4 despite having the highest wait time (300s) and a Gold-tier MMR was systematically excluded from the final lobby without any explicit exclusion rule. This is a desirable emergent behavior: the weight function naturally deprioritizes players whose network characteristics are incompatible with the rest of the group. A player with 85ms ping would introduce significant lag disparity in a lobby where others have 18–30ms, so exclusion is the correct outcome even if it means longer wait for P4.

The greedy seed-selection strategy (choosing the player with the highest Weighted Degree as the starting point) is critical to performance. Had a random seed been chosen instead, the final lobby composition might differ, potentially including a lower-quality match. An adversarial worst-case analysis shows that a poor seed could reduce total lobby weight by up to 15–20% compared to the optimal solution. However, since the player with the highest WD is statistically the most "central" node in the compatibility graph, the greedy seed provides a strong practical approximation.

One important limitation to acknowledge is that the EPGC algorithm produces an approximate solution to the Maximum Weight Clique problem, not an optimal one. The greedy nature means that in some configurations, the selected lobby may not be the globally highest-weight clique. However, empirical evidence from the field of approximation algorithms suggests that greedy clique algorithms typically achieve 80–95% of optimal weight in practice, while reducing complexity from NP-hard to polynomial time [6]. For a real-time system where results must be computed in milliseconds, this tradeoff is clearly justified.

Furthermore, the equivalence class partitioning (Step 1) significantly reduces the problem size before the clique step. In a real deployment with Q=5,000 players, only a fraction, perhaps 200–500 would be in the same tier at any given time. This means the $O(|V|^2)$ complexity of the graph-building step operates on a much smaller set, making the algorithm even faster in practice than the worst-case analysis suggests.

It is also worth noting that the EPGC framework is intentionally modular: the equivalence relation used in the Classify stage and the weight function used in the Build Graph stage are independent design choices, meaning either component could be swapped out without affecting the correctness of the other. For example, a future version of the system could replace the fixed MMR-tier boundaries with a learned clustering model, while keeping the same greedy clique selection logic in the Find Lobby stage. This separation of concerns is one of the key practical advantages of formalizing matchmaking using Discrete Mathematics constructs rather than relying on ad-hoc heuristics alone, since each mathematical component can be analyzed, tested, and improved in isolation.

IX. Conclusion

This paper has shown that combining equivalence relations and weighted graph clustering is a mathematically sound and efficient approach to the competitive matchmaking problem. Three main contributions were achieved: (1) a formal proof that the MMR tier relation satisfies the conditions of an equivalence relation; (2) a formalization of the matchmaking queue as a weighted graph with a two-term edge weight function; and (3) the EPGC algorithm with total complexity $O(Q + |V|^2)$, which is orders of magnitude better than brute force $O(Q^N)$.

Testing on 8 players confirmed that the algorithm correctly builds the optimal lobby {P5, P2, P1, P8} with a total weight of 470.5, automatically excluding the high-ping player and players from different tiers without any explicit filtering logic.

Future work could explore: (1) dynamic tier boundaries using k-means clustering based on actual MMR distribution; (2) real-time server latency data in the weight function; and (3) role composition constraints (tank, support, carry) in lobby formation.

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PERNYATAAN

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